**2.4 Cumulative Distribution Function (CDF)**

The cumulative distribution function of random variable  is



For any real number *x*, the CDF is the **probability** that the random variable *X* is no larger than *x*.

For all 



: Subscript corresponds to the **name of the random variable**

Properties of Cumulative Distribution Function (CDF)

For any discrete random variable  with range  satisfying,

a) .

b) For all.

c) For, and arbitrarily small positive number



d)  for all  such that 

Ex 2.23] Random variable  and PMF given as



Plot PMF and corresponding CDF





Ex 2.24] Let’s consider the geometric  random variable, with PMF given below



What is cumulative distribution function (CDF)?

1. when 
2. when 





 then

Let ,



For this kind problem, the geometric series can be used to get solutions.



Q 2.4] Use CDF to find the following probabilities and draw the PMF plot



1. *P*[*Y <* 1] = *FY* (1−)= 0
2. *P*[*Y* ≤ 1] = *FY* (1)= 0*.*6
3. *P*[*Y >* 2] = 1 − *P*[*Y* ≤ 2] = 1 − *FY* (2)= 1 − 0*.*8 = 0*.*2

(4) *P*[*Y* ≥ 2] = 1 − *P*[*Y <* 2] = 1 − *FY* (2−)= 1 − 0*.*6 = 0*.*4

(5)*P*[*Y* = 1] = *P*[*Y* ≤ 1] − *P*[*Y <* 1] = *FY* (1+)− *FY* (1−)= 0*.*6

(6) *P*[*Y* = 3] = *P*[*Y* ≤ 3] − *P*[*Y <* 3] = *FY* (3+)− *FY* (3−)= 0*.*8 − 0*.*8 = 0

**2.5 Averages**

Mean, Median, and Mode

Ex] 9 5 10 8 4 7 5 5 8 7

Median: 7

Mode: 5

Mean: 

* **Mean is Expected value.**
* **The expected value of**  **is defined as**

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The geometric (*p*) random variable *X* has expected vale 

Proof: Let. The PMF of *X* becomes





*Math fact B.6 in page 508*

If , 

So,



For discrete **uniform (*k, l*) random** variable *X,* the mean value is

